Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced

Tuesday 15 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

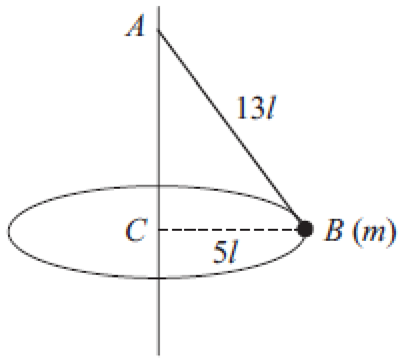

Figure 1

A garden game is played with a small ball B of mass m attached to one end of a light inextensible string of length $13l$. The other end of the string is fixed to a point A on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius $5l$ and centre C, where C is vertically below A. Modelling the ball as a particle, find

(a) the tension in the string,

(3)

(b) the speed of the ball.

(4)

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2. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P. The magnitude of this force is inversely proportional to x^2.

At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a sphere of radius R.

(a) Prove that the magnitude of the gravitational force on P is $\dfrac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U.

(b) Find U in terms of g and R.

(7)

3.

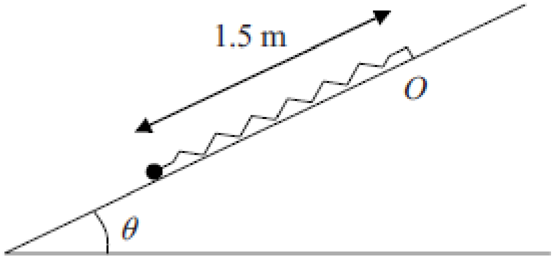

Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(9)

4.

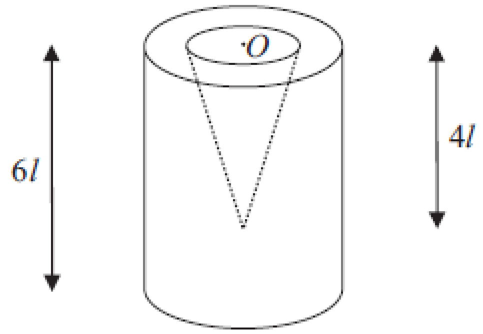

Figure 3

A container is formed by removing a right circular solid cone of height $4l$ from a uniform solid right circular cylinder of height $6l$. The centre O of the plane face of the cone coincides with the centre of a plane face of the cylinder and the axis of the cone coincides with the axis of the cylinder, as shown in Figure 3. The cylinder has radius $2l$ and the base of the cone has radius l.

(a) Find the distance of the centre of mass of the container from O.

(6)

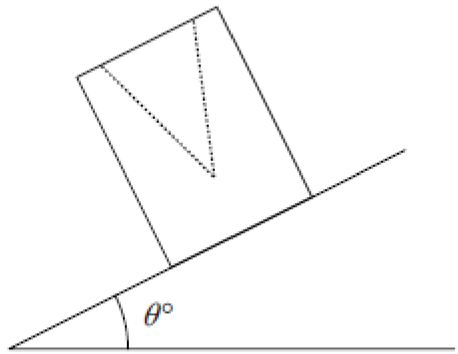

Figure 4

The container is placed on a plane which is inclined at an angle $\theta°$ to the horizontal. The open face is uppermost, as shown in Figure 4. The plane is sufficiently rough to prevent the container from sliding. The container is on the point of toppling.

(b) Find the value of θ.

(4)

5.

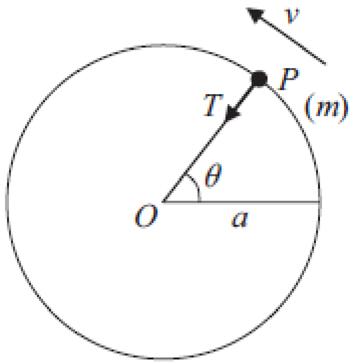

Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at the point O. The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u, where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T, as shown in Figure 5.

(*a*) Find, in terms of a, g and θ, an expression for v^2.

(3)

(*b*) Find, in terms of m, g and θ, an expression for T.

(4)

(*c*) Prove that P moves in a complete circle.

(3)

(*d*) Find the maximum speed of P.

(2)

6. At time $t = 0$, a particle P is at the origin O moving with speed 2 m s^{-1} along the x-axis in the positive x-direction. At time t seconds ($t > 0$), the acceleration of P has magnitude $\dfrac{3}{(t+1)^2}$ and is directed towards O.

(a) Show that at time t seconds the velocity of P is $\left(\dfrac{3}{t+1} - 1\right)$ m s^{-1}.

(5)

(b) Find, to 3 significant figures, the distance of P from O when P is instantaneously at rest.

(7)

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7. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A. A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O, vertically below A.

(a) Find the distance AO.

(3)

The particle is now raised to point C vertically below A, where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\dfrac{a}{g}}$.

(5)

It is given that $1OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.

(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D, moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.

(4)

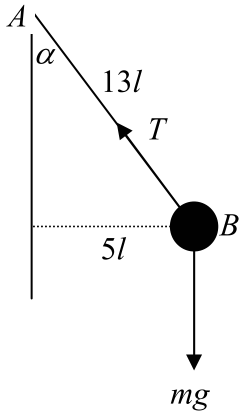
TOTAL FOR PAPER: 75 MARKS

END

edexcel

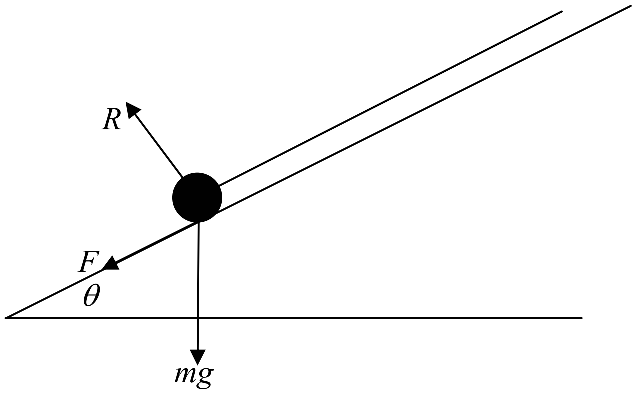
Summer 2010
Mechanics M3 6679
Mark Scheme

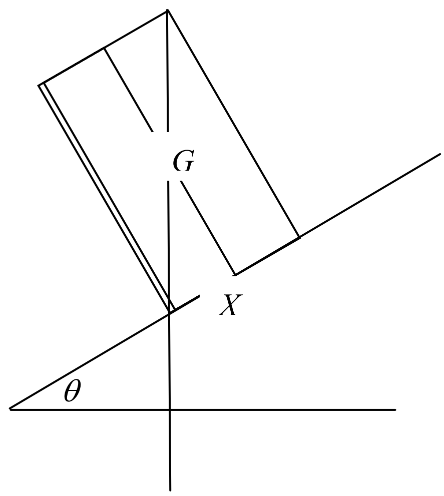
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q1

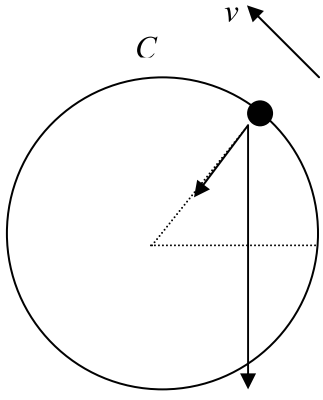
(a)</td><td>

$\cos\alpha = \frac{12}{13}$
$R(\uparrow)\quad T\cos\alpha = mg$
$T \times \frac{12}{13} = mg$
$T = \frac{13}{12}mg$ oe</td><td>B1
M1
A1 (3)</td></tr>
<tr><td>(b)</td><td>Eqn of motion $T\sin\alpha = m\frac{v^2}{5l}$
$\frac{13mg}{12} \times \frac{5}{13} = m\frac{v^2}{5l}$
$v^2 = \frac{25gl}{12}$
$v = \frac{5}{2}\sqrt{\frac{gl}{3}}$ $\left(\text{accept } 5\sqrt{\frac{gl}{12}} \text{ or } \sqrt{\frac{25gl}{12}} \text{ or any other equiv}\right)$</td><td>M1 A1
M1 dep
A1 (4)
[7]</td></tr>
</table>

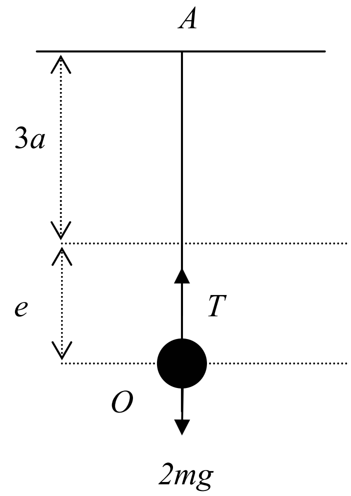
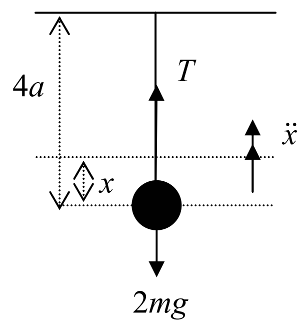
Question Number	Scheme	Marks
Q2 (a)	$F = (-)\dfrac{k}{x^2}$	M1
	$mg = (-)\dfrac{k}{R^2}$	M1
	$F = \dfrac{mgR^2}{x^2}$ *	A1 (3)
(b)	$m\ddot{x} = -\dfrac{mgR^2}{x^2}$	M1
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{gR^2}{x^2}$	M1
	$\dfrac{1}{2}v^2 = \int\left(-\dfrac{gR^2}{x^2}\right)\mathrm{d}x$	M1 dep on 1st M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} \quad (+c)$	A1
	$x = R,\ \ v = 3U \qquad \dfrac{9U^2}{2} = gR + c$	M1 dep on 3rd M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} + \dfrac{9U^2}{2} - gR$	
	$x = 2R,\ v = U \qquad \dfrac{1}{2}U^2 = \dfrac{gR^2}{2R} + \dfrac{9U^2}{2} - gR$	M1 dep on 3rd M mark
	$U^2 = \dfrac{gR}{8}$	
	$U = \sqrt{\dfrac{gR}{8}}$	A1 (7)
		[10]

Question Number	Scheme	Marks
Q3		
	EPE lost $=\frac{\lambda\times 0.6^2}{2\times 0.9}-\frac{\lambda\times 0.1^2}{2\times 0.9}\left(=\frac{7}{36}\lambda\right)$	M1 A1
	R$(\uparrow)$ $R=mg\cos\theta$	M1
	$=0.5g\times\frac{4}{5}=0.4g$	
	$F=\mu R=0.15\times 0.4g$	M1 A1
	P.E. gained = E.P.E. lost − work done against friction	
	$0.5\text{g}\times 0.7\sin\theta=\frac{\lambda\times 0.6^2}{2\times 0.9}-\frac{\lambda\times 0.1^2}{2\times 0.9}-0.15\times 0.4g\times 0.7$	M1 A1 A1
	$0.1944\lambda=0.5\times 9.8\times 0.7\times\frac{3}{5}+0.15\times 0.4\times 9.8\times 0.7$	
	$\lambda=12.70......$	
	$\lambda=13\text{ N}$ or 12.7	A1
		[9]

Question Number	Scheme	Marks
Q4 (a)	<table><tr><td></td><td>cone</td><td>container</td><td>cylinder</td></tr><tr><td>mass ratio</td><td>$\frac{4\pi l^3}{3}$</td><td>$\frac{68\pi l^3}{3}$</td><td>$24\pi l^3$</td></tr><tr><td></td><td>4</td><td>68</td><td>72</td></tr><tr><td>dist from O</td><td>l</td><td>$\bar{x}$</td><td>$3l$</td></tr></table>	M1 A1 B1
	Moments: $4l + 68\bar{x} = 72 \times 3l$	M1 A1ft
	$\bar{x} = \frac{212l}{68} = \frac{53}{17}l$ accept $3.12l$	A1 (6)
(b)	 $GX = 6l - \bar{x}$ seen	M1
	$\tan\theta = \frac{2l}{6l - \bar{x}}$	M1 A1
	$= \frac{2 \times 17}{49}$	
	$\theta = 34.75... = 34.8$ or 35	A1 (4) **[10]**

Question Number	Scheme	Marks
Q5		
(a)	Energy: $mga\sin\theta = \frac{1}{2}m\times 5ag - \frac{1}{2}mv^2$ $v^2 = 5ag - 2ag\sin\theta$	M1 A1 A1 (3)
(b)	Eqn of motion along radius: $T + mg\sin\theta = \frac{mv^2}{a}$ $T = \frac{m}{a}\left(5ag - 2ag\sin\theta\right) - mg\sin\theta$ $T = mg\left(5 - 3\sin\theta\right)$	M1 A1 M1 A1 (4)
(c)	At C, $\theta = 90°$ $T = mg\left(5-3\right) = 2mg$ $T > 0 \quad \therefore P$ reaches C	M1 A1 A1 (3)
(d)	Max speed at lowest point ($\theta = 270°$; $v^2 = 5ag - 2ag\sin 270$) $v^2 = 5ag + 2ag$ $v = \sqrt{(7ag)}$	M1 A1 (2) **[12]**

Question Number	Scheme	Marks
Q6 (a)	$\dfrac{d^2x}{dt^2} = -\dfrac{3}{(t+1)^2}$	M1
	$\dfrac{dx}{dt} = \int -3(t+1)^{-2}\,dt$	
	$= 3(t+1)^{-1}\ (+c)$	M1 A1
	$t=0,\ v=2 \quad 2=3+c \quad c=-1$	M1
	$\dfrac{dx}{dt} = \dfrac{3}{t+1} - 1$ *	A1 (5)
(b)	$x = \int\left(\dfrac{3}{t+1} - 1\right)dt$	M1
	$= 3\ln(t+1) - t \quad (+c')$	A1
	$t=0,\ x=0 \quad \Rightarrow c' = 0$	B1
	$x = 3\ln(t+1) - t$	
	$v = 0 \Rightarrow \dfrac{3}{t+1} = 1$	M1
	$t = 2$	A1
	$x = 3\ln 3 - 2$	M1
	$= 1.295...$	
	$= 1.30$ m (Allow 1.3)	A1 (7) **[12]**

Question Number	Scheme	Marks
Q7	 	
(a)	R$(\uparrow)$ $T = 2mg$	B1
	Hooke's law: $T = \dfrac{6mge}{3a}$	
	$2mg = \dfrac{6mge}{3a}$	M!
	$e = a$	
	$AO = 4a$	A1 (3)
(b)	 	
	H.L. $T = \dfrac{6mg(a-x)}{3a} = \dfrac{2mg(a-x)}{a}$	B1ft
	Eqn. of motion $-2mg + T = 2m\ddot{x}$	M1
	$-2mg + \dfrac{2mg(a-x)}{a} = 2m\ddot{x}$	M1
	$-\dfrac{2mgx}{a} = 2m\ddot{x}$	
	$\ddot{x} = -\dfrac{g}{a}x$	A1
	period $2\pi\sqrt{\dfrac{a}{g}}$ *	A1 (5)

Question Number	Scheme	Marks
(c)	$v^2 = \omega^2\left(a^2 - x^2\right)$ $v_{\max}{}^2 = \frac{g}{a}\left(\left(\frac{a}{4}\right)^2 - 0\right)$ $v_{\max} = \frac{1}{4}\sqrt{(ga)}$	 M1 A1 A1 (3)
(d)	$x = -\frac{a}{8}$ $\quad v^2 = \frac{g}{a}\left(\frac{a^2}{16} - \frac{a^2}{64}\right)$ $= \frac{3ag}{64}$ $v^2 = u^2 + 2as$ $0 = \frac{3ag}{64} - 2gh$ $h = \frac{3a}{128}$ Total height above $O = \frac{a}{8} + \frac{3a}{128} = \frac{19a}{128}$	M1 M1 A1 A1 (4) **[15]**